Forecasting Accuracy of Holt-Winters Exponential Smoothing: Evidence From New Zealand

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Abstract: Financial time series is volatile, dynamic, nonlinear, nonparametric, and chaotic. Accurate forecasting of stock market prices and indices is always challenging and complex endeavour in time series analysis. Accurate predictions of stock market price movements could bring benefits to different types of investors and other stakeholders to make the right trading strategies.

Adopting a technical analysis perspective, this study examines the predictive power of Holt-Winters Exponential Smoothing (HWES) methodology by testing the models on the New Zealand stock market (S&P/NZX50) Index. Daily time-series data ranging from January 2009 to December 2017 are used in this study. The forecasting performance of the investigated models is evaluated using the root mean square error (RMSE], mean absolute error (MAE) and mean absolute percentage error (MAPE).

Employing HWES on the undifferenced S&P/NZX50 Index (model 1) and HWES on the differenced S&P/NZX50 Index (model 2) we find that model 1 is the superior predictive algorithm for the experimental dataset. When the tested models are evaluated overtime of the sample period we find the supportive evidence to our original findings. The evaluated HWES models could be employed effectively to predict the time series of other stock markets or the same index for diverse periods (windows) if substantiate algorithm training is carried out.

1. INTRODUCTION

Financial markets are influenced by a variety of interdependent determinants. Adopting the fundamental perspective, Fama (1991), Chen, Roll, and Ross (1986), Chen (1991), Granger (1986), Engle and Granger (1987), Kwon and Shin (1999), Wongbangpo and Sharma (2002), Dassanayake and Jayawardena (2017) and so on evaluated the deterministic factors of the stock market price movements. They found the sources are multifaceted and originating from numerous sources ranging from domestic to international economic environments, motivation, and psychology of individual and institutional investors, local and international political situations, the degree of integration with international markets, and the impact of spontaneous events. These multidimensional forces consistently impacting on the stock market price movements in dissimilar scales thereby creating these time series to be dynamic, highly volatile, nonlinear, nonparametric, and turbulent.

Researchers and practitioners have been devoted to forecasting future trends in financial time series using different statistical, soft computing, and hybrid methods. These attempts could be broadly classified into technical analysis and fundamental analysis where these two schools of thought are at the opposite ends of the spectrum in devising forecasting models and taking different methodological, philosophical, and conceptual approaches. A time series can be defined as "*a set of regular time-ordered observations of a quantitative characteristic of an individual or collective phenomenon taken at successive, in most cases equidistant, periods of time*" (Statistics, O. E. C. D., 2013). Using this definition, a univariate time series is characterized by a vector of $y = [y_1, y_2,.., y_n]^T$, where y_t refers to the value of y in time step t and n refers to the total number of observations.

Fundamental analysts strive to determine the intrinsic value of an asset (company, industry, investment) based on the overall macroeconomic conditions, the management strategies of the company, industry environment, and its political atmosphere. Thus, the fundamentalists employ numerical information about macroeconomic, financial, and other related factors to predict the perceived value of the asset. The fundamental approach could be company-specific, industry-specific, or the economy as a whole. The technical analysis, on the other hand, has full reliance on the historical values of the time series to capture the past trends and cycles. Utilising charts, statistical and soft computing techniques, the technicians develop models to predict either high frequency or low-frequency financial time series. Schwager (1993, 1995); Covel (2004) uncovered that most brokerage companies hedge funds in practice have been heavily reliant on the technical analysis than the fundamental methodology.

The primary objective of this research is to evaluate the effectiveness and the performance of Holt-Winters Exponential Smoothing (HWES) for predicting the New Zealand stock market (S&P/NZX50) index. This paper contributes to the limited technical analysis based literature applied to the New Zealand stock market price/index prediction domain.

The remainder of the paper is organised as follows. Section 2 reviews the previous literature on stock market price-prediction models. In section 3 we review the Holt-Winters Exponential Smoothing (HWES) methodology applied in this study. Data and sample description is provided in section 4. Experimental results are reported in section 5. Section 6 contains the conclusions with some limitations and future research directions.

2. LITERATURE REVIEW

2.1 Preamble

The forecasting methodology of exponential smoothing (ES) appears to be originated from Robert G. Brown in 1944 when he was working as an operations research analyst in the US Navy during the second world war. Using a continuous set of data, he essentially applied an exponentially weighted moving average methodology to develop a model for tracking the velocity and angles of the enemy submarines. In the early 1950s, Brown extended the model from a continuous-time series to discrete data and improved it to deal with trends and seasonal fluctuations. Brown's application in predicting the demand for spare parts of the US Navy inventory system was vastly successful in terms of forecasting accuracy, thus, the methodology was implemented by the US Navy Inventory System (Gardner, 2006). In 1956, Brown presented the work of ES of inventory demands at a conference of Operations Research Society of America, and subsequently, this presentation established the basis of his first book, *Statistical forecasting for inventory control* (Brown, 1959). The general ES methodology was presented in Brown's second book, *Smoothing, Forecasting, and Prediction of Discrete Time Series* (Brown, 1963). Holt (1957) worked independently of Brown to formulate an alternative method for smoothing seasonal data whilst adopting a similar method for smoothing additive trends. Holt's original work was documented in the US Office of Naval Research (ONR) memorandum (Holt, 1957) but went unpublished until 2004 when it got published in the *International Journal of Forecasting* (Holt, 2004). Holt's additive and multiplicative seasonal exponential smoothing methodology gained wide publicity with the work of Winters (1960) where Winters empirically tested Holt's methods. Thus, Holt's seasonal versions are known as Holt-Winters' forecasting methods. Further development and collaborations to Holt's models were made by Muth (1960), Pegels (1969), Holt, Modigliani, Muth, & Simon (1960). Hyndman, Koehler, Snyder & Grose (2002) advocated a new approach for the categorization of ES methods. In a broader context, the popular ES methods are simple exponential smoothing (SES), Holt's linear method (additive trend, no seasonality), Holt-Winters' additive method (additive trend, additive seasonality), and Holt-Winters' multiplicative method (additive trend, multiplicative seasonality).

2.2 Variations

Many variations to the original ES have been projected. Rosas and Guerrero (1994) evaluated the incorporation of additional information through one or more constraints in exponential smoothing forecasts. They proposed to accommodate them as linear restrictions and suggested that appropriate use of such information improves prediction accuracy and precision. Carreno & Madinaveitia (1990) aimed at establishing the announced price increases through an adjustment to sales plus exponential smoothing, moving indices to normalize the original sales data, and modification of the forecast. Their rationale is useful for time series forecasting in an economy with high inflation. Williams & Miller (1999) proposed a methodology for letting the predictor incorporate the judgmental adjustments *within* the exponential smoothing model. This study demonstrated the proposed model is better than the alternative models tested. Lawton (1998) explored the precision of the Additive Holt-Winters methodology and argued for renormalization of the seasonal indices at each period, as it removes bias in estimates of the level and seasonal components. Roberts (1982) and McKenzie (1986) proposed marginally different normalization schemes to Lawton (1998). Later, Archibald and Koehler (2003) developed innovative and much simpler renormalization equations arriving with similar forecasts. SES with drift is an important variation in between SES and Holt's method which is equivalent to Holt's method setting the trend parameter to be zero. Hyndman & Billah (2003) exhibited that Theta method proposed by Assimakopoulos & Nikolopoulos (2000) is simply a special case of SES with drift.

2.3 Application of ES to Financial market prediction

Exponential Smoothing (ES) is a simple yet robust methodology in time series prediction. ES can be applied to time series that exhibit homoscedastic as well as heteroscedastic patterns. Although the homoscedastic case is similar to the ARIMA process, the heteroscedastic case is different from the ARIMA process. Thus, Ord, Koehler & Snyder (1997) argued that ES could be expanded beyond the ARIMA class. Leung, Daouk & Chen (2000) tested the predictive power of two types of models on the S&P 500, FTSE 100, and Nikkei 225 indices. The tested classification models predict direction based on probability, include linear discriminant analysis, logit, probit, and probabilistic neural network. The tested level estimation counterparts are exponential smoothing, multivariate transfer function, vector autoregression with Kalman filter, and multilayered feedforward neural network. The empirical investigation finds that the classification models performed better than the level estimation models in terms of forecasting the direction of the stock market movement and maximising returns from investment trading. Maris, Pantou, Nikolopoulos, Pagourtzi & Assimakopoulos (2004) evaluated the forecasting performance of ES, random walk (RW), and four models of ARCH family employing MAPE and RMSE as the performance criteria. Applying the models to the Greek FTSE/ASE 20 stock index, they found RW outperformed the rest of the models tested. Taylor (2004) tested the forecasting capabilities of smooth transition exponential smoothing (STES) and a verity of GARCH models for the S&P500 index. Employing RMSE as the performance evaluation criteria and he found that STES was a better forecasting model for the tested sample. Pereira (2004) examined the forecasting performance of RW, ES, ARCH, and so on using MSE, RMSE, and MAPE as the evaluation criteria. Applying the analysis to the Portuguese stock market, he found that the superiority of the ARCH model. Poon, Hyung & Granger (2006) used ES, random walk, fractional integrated (FI) break, GARCH, and so on to test the forecast performance of the daily volatility of the S&P500 index. Using MAE as the evaluation criterion they found that FI was the superior forecasting model for 10 days or beyond. Using ES, exponentially weighted moving average (ESWA), ARCH/GARCH, and so on Balaban, Bayar & Faff (2006) tested the accuracy of the prediction models. Mean absolute error, root mean squared error and mean absolute percentage error were used as the performance criteria. Daily stock market indices of 15 countries were tested and the ES model performed better than the rest of the tested models. Bley & Olson (2008) used ES, single-factor, and multifactor volatility index models, GARCH, and so on to forecast the volatility of the S&P100, S&P500, and NASDAQ100 indices. Using RMSE, MAE, etc., they found that the single-factor and multifactor volatility index and ES are the best foresting models. Using traditional time series decomposition (TSD), Holt/Winters (H/W) models, Box-Jenkins (B/J) methodology, and neural network (NN) models, Tseng, Kwon & Tjing (2012) analysed daily closing stock prices of 50 randomly selected stocks during 1998 to 2010. MAPE was used to determine the foresting accuracy and they found B/J, H/W, and normalised NN models are superior in comparison to TSD and non-normalised NN models. Awajan, Ismail & Wadi (2018) analysed stock market data of 6 countries to determine the forecasting performances of Holt-Winter method, ARIMA models, Structural Time Series, Theta method, Exponential smoothing state space method (ETS), Random Walk method (RW) and hybrid EMD-HW with (without) bagging methods. RMSE, MAE, MAPE, MASE TheilU performance criteria were used and they found that the EMD-HW bagging model is more accurate in comparison to the other tested models. Sharif and Hasan (2019) applied Holt's method on the time series of Dhaka Stock Exchange and found the suitability of different smoothing constants for prediction accuracies.

2.4 Application to New Zealand stock market

Application of the ES model to forecast the time series of New Zealand financial markets is limited. Yu (2002) evaluated the performance of nine alternative models [random walk, historical average, moving average, simple regression, exponential smoothing, exponentially-weighted moving average (EMA), autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroskedasticity (GARCH) and stochastic volatility (SV)] for predicting the volatility in the New Zealand stock market. Using RMSE, MAE, and Theil-U evaluation measures, they analysed the daily data of the NZSE40 capital index for 1980 to 1998 to forecast the monthly stock market volatility. The exponential smoothing method was adjudged the best model based on the MAE whilst SV model outperformed the others based on both RMSE and Theil-U.

3. METHODOLOGY

3.1 Exponential Smoothing

The key characteristic of Exponential smoothing (ES) is that the predictions are weighted combinations of the past values of the time series, with more recent observations are assigned with relatively higher weight than the older observations. As the name reflects, the weights in the ES method decay exponentially as the observations get older. The smoothing scheme could be a single ES, double ES, and triple ES. The triple ES is also known as Holt-Winters ES (HWES).

The HWES method is a robust yet easy to use forecasting methodology which works quite well with real-world time series for short-term predictions. Thus, we use the HWES methodology in our study. The HWES methodology is summarised in equations 1- 4.

Let the overall smoothed level of the time series, the smoothed multiplicative trend, and the smoothed seasonal index at the time t are denoted by l_t, b_t, and s_t respectively. The formulae for updating l_t, b_t, and s_t, when a new observation y_t, becomes available, are given in equations 1- 4. Let α, β and γ denote the smoothing parameters for updating the level, trend, and seasonal index respectively whilst m denotes the number of observations per seasonal cycle.

$$l_t = \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)\left(l_{t-1} + b_{t-1}\right) \qquad \textit{Overall Smoothing} \qquad (1)$$

$$b_t = \beta\left(l_t - l_{t-1}\right) + (1-\beta) b_{t-1} \qquad \textit{Trend Smoothing} \qquad (2)$$

$$s_t = \gamma \frac{y_t}{\left(l_{t-1} + b_{t-1}\right)} + (1-\gamma) s_{t-m} \qquad \textit{Seasonal Smoothing} \qquad (3)$$

$$\hat{y}_{t+h/t} = \left(l_t + hb_t\right) s_{t+h-m(k+1)} \qquad \textit{Forecast} \qquad (4)$$

Where y_t is the observed value of the time series in period t; l_t is the smoothed level of the series computed after y_t is observed; b_t is the smoothed multiplicative trend at the end of period t; s_t is the smoothed seasonal index at the end of period t; $\hat{Y}_{t+h/t}$ refers to forecast for h/t periods ahead from origin t. α, β and γ are the constants of Holt-Winters ES model. The smoothing parameters and initial estimates for the elements are estimated by minimising the associated errors through performance evaluation statistics. Also, for model identification, Akaike Information Criterion (AIC) will be used (Akaike, 1973; Faraway & Chatfield, 1998; Kihoro, Otieno & Wafula, 2004).

3.2 Performance Evaluation

Three error statistics are used to evaluate the performance of the models tested. They are mean absolute error (MAE), mean absolute percentage error (MAPE) and root mean square error (RMSE). These error statistics are given below:

$$MAE = \frac{1}{N} \sum_{i=1}^{N} \left|X_t - \widehat{X}_t\right| \qquad (5)$$

$$MAPE = \frac{1}{N} \sum_{t=1}^{N} \left|\frac{X_t - \widehat{X}_t}{\widehat{X}_t}\right| \qquad (6)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(X_t - \widehat{X}_t\right)^2} \qquad (7)$$

However, in an event where inconsistent conclusions transpire from these criteria, the MAPE, suggested by Makridakis (1993), is used as the benchmark as MAPE is relatively more stable than other criteria from a theoretical and practical viewpoint. For model identification, the AIC is used (Akaike [27]). AIC is outlined below.

$$AIC(p) = n\, Ln\left(\hat{\sigma}_e^2\right) + 2p \qquad (8)$$

4. DATA AND SAMPLE DESCRIPTION

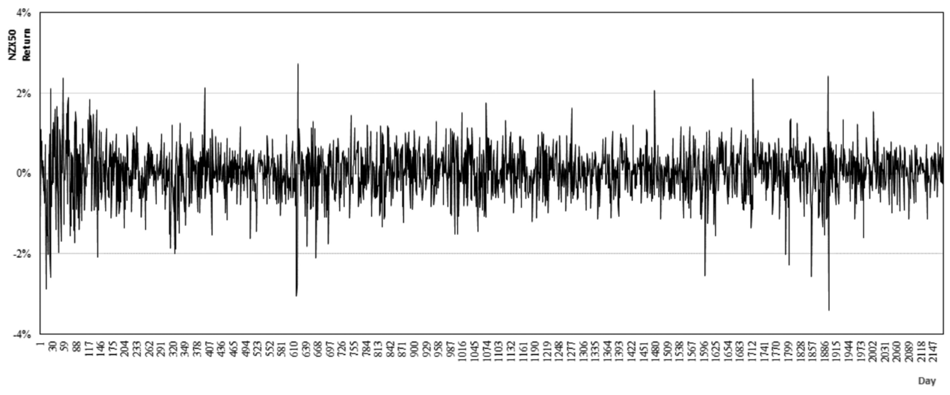
S&P/NZX50 Index data are extracted from S&P Dow Jones Indices produced by S&P Global (S&P/NZX50 Index, 2018). The index is developed to capture the overall performance of the 50 largest stocks listed on the Main Board (NZSX) of New Zealand's Exchange (NZX). We use daily price series of S&P/NZX50 Index from 2009 to 2017 having a total number of 2173 observations. Continuously compounded daily returns are generated using the formula $R_t = l_n\ (P_t/P/_{t-1})$ where P_t and R_t refer to the price of the S&P/NZX50 Index and continuously compounded return on trading day t respectively. The data range was split into training and test sets. To evaluate the performance of each model configured, the first 1500 observations (approximately 70%) are used as the training sample and the rest of the observations are used for prediction purposes.

Figure 1 portrays the time plot of the S&P/NZX50 Index confirming the time series is nonstationary and exhibits an upward trend and some degree of seasonal, cyclical, and irregular variations.

Figure 1 Time plot of daily price series of S&P/NZX50 Index, 2009 - 2017

Figure 2 shows the time plot of the differenced S&P/NZX50 Index confirming the differenced series is stationary both in its mean and variance.

Figure 2 Time plot of daily difference price (return) series of S&P/NZX50 Index, 2009 - 2017



5. RESULTS

5.1 Tests for Stationarity

From a robustness perspective, two formal tests for stationarity are carried out. They are Augmented Dickey-Fuller (ADF) (Dickey and Fuller 1979, 1981) Phillips-Perron [Perron, 1987; Phillips & Perron (1988)] unit root tests. The Akaike Information Criteria (AIC) (Faraway & Chatfield 1998; Kihoro et al. 2004), is carried out to determine the optimum number of lags for ADF and Phillips-Perron tests. Both stationarity test results for S&P/NZX50 Index confirm that the price series is non-stationary at levels; however, the first difference of the price series (return) is stationary based on MacKinnon (1996) one-sided p-values at the 1% significance level (detailed results are available on request). These results confirm that the S&P/NZX50 Index is integrated of order 1. Also, the ADF and Phillips-Perron tests reinforce each other.

5.2 Comparison of Holt Winter's Exponential Smoothing (HWES) models

To determine the correct HWES model to forecast the S&P/NZX50 Index, the "HoltWinters()" function in R software is used. Technically HWES could have three components namely alpha (the smoothed level), beta (the smoothed time trend), and gamma (the smoothed seasonal component). Due to the non-trading days present in the index, the number of trading days per week (or month) is not going to be equal. If the seasonal smoother (gamma) is included when performing HWES in R software, the time series object it is applied to must have the frequency stated (with the frequency being at least 2). Due to the unequal numbers of trading days, we are unable to declare the frequency with accuracy and instead state a frequency of 1. To run the HWES, the seasonal smoothing component was omitted (leaving just the level and time trend). When the graphs of the data are examined carefully, we could not find a strong seasonal component which enables us to justify the decision of excluding the seasonal component. HWES was performed on the undifferenced NZX50 time series incorporating alpha and beta but excluding gamma (Model 1). HWES without beta or gamma was also performed on the differenced NZX50 time series. Each model was tested by comparing 1-ahead forecasts with the corresponding test data observations.

The performance comparisons of the tested HWES models are presented in Table 1.

Table 1

Table 1 Robustness evaluation of HWES models on S&P/NZX50 Index			
HWES Model	RMSE	MAE	MAPE
Model 1: HWES (alpha, beta) on NZX50 Index	18.4089	13.6894	0.0041781
Model 2: HWES (alpha) on difference of NZX50 Index	18.4091	13.6905	0.0041784

RMSE, MAE, and MAPE are used as performance evaluation criteria. Prediction performance results of model 1 [HWES (alpha, beta) on the NZX50 Index] shows its superiority over Model 2 tested [HWES (alpha) on the difference of NZX50 Index]. From an empirical perspective, the structure of model 1 makes more sense than the other model tested. Both models 1 and 2 refitted for each test predicted.

To determine the robustness of the investigation, we also evaluate how both HWES models perform over time. Approximately 70% of each year's data are used to train the models and the rest is used for prediction purposes. We apply the same performance evaluation criteria and the results are presented in Table 2.

Table 2

Table 2 Robustness evaluation of the tested HWES models on NZX50 Index over time										
Model 1: HWES (alpha, beta) on NZX50 Index										
	2009	2010	2011	2012	2013	2014	2015	2016	2017	**Average**
RMSE	12.2766	10.1173	12.8739	11.5958	13.4242	13.3414	15.3209	24.7211	13.3088	14.10890
MAE	10.1207	8.0740	10.1788	9.4989	10.6626	10.4930	12.8699	17.5289	10.3177	11.08272
MAPE	0.0048	0.0039	0.0051	0.0042	0.0041	0.0037	0.0044	0.0053	0.0028	0.00425
Model 2: HWES (alpha, beta) on NZX50 Index										
	2009	2010	2011	2012	2013	2014	2015	2016	2017	**Average**
RMSE	12.2753	10.0600	12.8741	11.5957	13.4129	13.3415	15.3108	24.8717	13.3079	14.11666
MAE	10.1192	8.1021	10.1795	9.4988	10.6452	10.4932	12.8576	17.6392	10.3110	11.09396
MAPE	0.0048	0.0039	0.0051	0.0042	0.0041	0.0037	0.0043	0.0053	0.0028	0.00426

Table two shows the annual performance evaluation statistics of the two HWES models. Although mixed results are observable when the two models are evaluated over time, the average of the annual results suggests the predictive superiority of the model 1 [HWES (alpha, beta) on NZX50 Index] over model 2 [HWES (alpha) on the difference of NZX50 Index] justifying our original finding.

5.3 Forecast based on the best HWES Model

The model 1 [HWES (alpha, beta) on NZX50 Index], which is adjudged as the superior model, is used for prediction purposes. From a total number of 2,173 observations, 1,500 are used to train the model and the remainder is used for prediction purposes. Predictions are made for the period 30/03/2015 to 29/12/2017, a total of 672. The actual S&P/NZX50 Index values, the predicted values, and the associated residuals are presented in Table 3.

Table 3

Table 3 Model 1: HWES (alpha, beta) on NZX50 Index Actual vs Prediction											
Observation	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**
30/03/2015	2992.54	3009.61	-17.07	7/05/2015	2942.52	2959.01	-16.49	16/06/2015	2961.91	2964.2	-2.29
31/03/2015	2996.81	2991.59	5.22	8/05/2015	2944.8	2941.97	2.83	17/06/2015	2942.24	2962.86	-20.62
1/04/2015	2997.63	2997.96	-0.33	11/05/2015	2951.27	2943.4	7.87	18/06/2015	2927.19	2941.01	-13.82
2/04/2015	2995.48	2997.92	-2.44	12/05/2015	2950.39	2951.95	-1.56	19/06/2015	2943.51	2927.51	16.00
7/04/2015	3007.83	2996.67	11.16	13/05/2015	2952.01	2950.75	1.26	22/06/2015	2938.56	2941.48	-2.92
8/04/2015	3009.47	3009.06	0.41	14/05/2015	2944.84	2950.99	-6.15	23/06/2015	2938.61	2936.66	1.95
9/04/2015	3003.03	3010.78	-7.75	15/05/2015	2956.12	2945.28	10.84	24/06/2015	2940.31	2938.08	2.23
10/04/2015	3003.13	3003.99	-0.86	18/05/2015	2961.87	2955.11	6.76	25/06/2015	2918.83	2938.91	-20.08
13/04/2015	3006.7	3004.35	2.35	19/05/2015	2953.9	2962.35	-8.45	26/06/2015	2930.11	2919.53	10.58
14/04/2015	3020.98	3007.92	13.06	20/05/2015	2951.41	2953.5	-2.09	29/06/2015	2904.84	2930.6	-25.76
15/04/2015	3007.61	3022.02	-14.41	21/05/2015	2958.32	2950.11	8.21	30/06/2015	2915.61	2905.41	10.20
16/04/2015	3020.79	3008.05	12.74	22/05/2015	2959.95	2958.33	1.62	1/07/2015	2949.92	2915.6	34.32
17/04/2015	3010.38	3021.87	-11.49	26/05/2015	2970.12	2959.23	10.89	2/07/2015	2973.9	2949.54	24.36
20/04/2015	2991.27	3010.82	-19.55	27/05/2015	2950.03	2970.6	-20.57	6/07/2015	2940.89	2974.28	-33.39
21/04/2015	2987.8	2988.84	-1.04	28/05/2015	2960.12	2948.8	11.32	7/07/2015	2954.41	2940.05	14.36
22/04/2015	2975.53	2985.85	-10.32	29/05/2015	2994.61	2960.6	34.01	8/07/2015	2935.6	2954.13	-18.53
23/04/2015	2957.19	2976.52	-19.33	2/06/2015	3003.27	2995.23	8.04	9/07/2015	2920.2	2934.49	-14.29
24/04/2015	2961.01	2953.95	7.06	3/06/2015	2996.36	3004.61	-8.25	10/07/2015	2914.04	2918.03	-3.99
28/04/2015	2963.22	2958.28	4.94	4/06/2015	2999.8	2997.04	2.76	13/07/2015	2904.55	2913.57	-9.02
29/04/2015	2948.41	2960.3	-11.89	5/06/2015	3001.06	3002.06	-1.00	14/07/2015	2927.04	2902.95	24.09
30/04/2015	2974.36	2948.66	25.70	9/06/2015	2989.85	3001.88	-12.03	15/07/2015	2955.07	2926.37	28.70
1/05/2015	2977.47	2975.37	2.10	10/06/2015	2956.78	2990.36	-33.58	16/07/2015	2964.1	2955.79	8.31
4/05/2015	2961.9	2977.23	-15.33	11/06/2015	2984.57	2956	28.57	17/07/2015	2979.17	2964.27	14.90
5/05/2015	2972.53	2959.6	12.93	12/06/2015	2978.74	2984.93	-6.19	20/07/2015	2983.32	2980.02	3.30
6/05/2015	2960.97	2971.32	-10.35	15/06/2015	2964.98	2978.75	-13.77	21/07/2015	2990.95	2985.16	5.79
22/07/2015	3016.82	2991.61	25.21	26/08/2015	2838.21	2852.52	-14.31	1/10/2015	2793.15	2796.91	-3.76
23/07/2015	3003.36	3018.04	-14.68	27/08/2015	2867.3	2833.9	33.40	2/10/2015	2797.19	2790.01	7.18
24/07/2015	2999.74	3004.18	-4.44	28/08/2015	2885.38	2867.85	17.53	5/10/2015	2815.71	2793.68	22.03
27/07/2015	2988.48	3000.49	-12.01	31/08/2015	2872.65	2885.47	-12.82	6/10/2015	2833.8	2812.93	20.87
28/07/2015	2976.43	2989.48	-13.05	1/09/2015	2872.01	2870.67	1.34	7/10/2015	2824.76	2832.93	-8.17
29/07/2015	2987.82	2976.74	11.08	2/09/2015	2836.56	2870.13	-33.57	8/10/2015	2812.64	2822.71	-10.07
30/07/2015	2998.55	2989.2	9.35	3/09/2015	2826.15	2834.43	-8.28	9/10/2015	2819.14	2810.98	8.16
31/07/2015	3013.36	3000.02	13.34	4/09/2015	2814.58	2821.77	-7.19	12/10/2015	2844.66	2817.7	26.96
3/08/2015	3032.14	3016.46	15.68	8/09/2015	2844.69	2814.52	30.17	13/10/2015	2851.15	2843.51	7.64
4/08/2015	3019.88	3035.53	-15.65	9/09/2015	2871.63	2844.49	27.14	14/10/2015	2863.31	2850.21	13.10

Table 3 continued

Table 3 *Continue* Model 1: HWES (alpha, beta) on NZX50 Index Actual vs Prediction											
Observation	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**
5/08/2015	3022.3	3022.38	-0.08	10/09/2015	2868.22	2870.81	-2.59	15/10/2015	2887.59	2864.26	23.33
6/08/2015	3017.3	3024.32	-7.02	11/09/2015	2856.71	2868.34	-11.63	16/10/2015	2909.74	2887.71	22.03
7/08/2015	2986.75	3019.02	-32.27	14/09/2015	2865.64	2857.13	8.51	19/10/2015	2917.15	2915.6	1.55
10/08/2015	2984.9	2987.12	-2.22	15/09/2015	2858.65	2864.53	-5.88	20/10/2015	2947.48	2917.44	30.04
11/08/2015	2963.18	2985.21	-22.03	16/09/2015	2861.37	2857.12	4.25	21/10/2015	2958.86	2947.97	10.89
12/08/2015	2930.03	2963.7	-33.67	17/09/2015	2874.63	2859.39	15.24	22/10/2015	2961.53	2962.08	-0.55
13/08/2015	2920.1	2927.51	-7.41	18/09/2015	2883.62	2874.26	9.36	23/10/2015	2985.06	2963.12	21.94
14/08/2015	2899.1	2919.41	-20.31	21/09/2015	2869.22	2882.86	-13.64	27/10/2015	3000.24	2989.91	10.33
17/08/2015	2914.87	2898.01	16.86	22/09/2015	2875.92	2868.09	7.83	28/10/2015	2999.22	3001.88	-2.66
18/08/2015	2906.39	2911.34	-4.95	23/09/2015	2839.8	2874.73	-34.93	29/10/2015	3001.22	3000.45	0.77
19/08/2015	2927.01	2905.89	21.12	24/09/2015	2849.65	2837.86	11.79	30/10/2015	2992.92	3002.97	-10.05
20/08/2015	2922.52	2926.08	-3.56	25/09/2015	2854.94	2847.53	7.41	2/11/2015	2991.65	2997.09	-5.44
21/08/2015	2926.45	2921.6	4.85	28/09/2015	2856.54	2853.18	3.36	3/11/2015	3010.72	2993.99	16.73
24/08/2015	2853.24	2926.43	-73.19	29/09/2015	2812.74	2855.05	-42.31	4/11/2015	3035.33	3012.37	22.96
25/08/2015	2856.28	2847.42	8.86	30/09/2015	2797.12	2809.57	-12.45	5/11/2015	3036.19	3036.86	-0.67
6/11/2015	3033.38	3039.11	-5.73	14/12/2015	3003.41	3021.45	-18.04	25/01/2016	3072.29	3046.51	25.78
9/11/2015	3022.46	3034.4	-11.94	15/12/2015	3006.04	3004.3	1.74	27/01/2016	3055.72	3073.61	-17.89
10/11/2015	2999.93	3023.53	-23.60	16/12/2015	3021.16	3006.74	14.42	28/01/2016	3059.58	3056.54	3.04
11/11/2015	3003.74	3000.7	3.04	17/12/2015	3029.82	3022.45	7.37	29/01/2016	3069.79	3060.66	9.13
12/11/2015	3008.6	3004.83	3.77	18/12/2015	3039.53	3030.62	8.91	1/02/2016	3071.92	3071.07	0.85
13/11/2015	2991.15	3009.76	-18.61	21/12/2015	3045.99	3041.12	4.87	2/02/2016	3074.7	3073.03	1.67
16/11/2015	2977.49	2991.96	-14.47	22/12/2015	3059.5	3047.15	12.35	3/02/2016	3051.46	3075.63	-24.17
17/11/2015	2980.45	2978.99	1.46	23/12/2015	3083.07	3062.22	20.85	4/02/2016	3053.61	3052.25	1.36
18/11/2015	2985.06	2981.62	3.44	24/12/2015	3098.1	3086.79	11.31	5/02/2016	3061.62	3054.4	7.22
19/11/2015	2992.77	2986.77	6.00	29/12/2015	3131.39	3100.77	30.62	9/02/2016	3020.58	3062.35	-41.77
20/11/2015	2999.4	2993.49	5.91	30/12/2015	3144.8	3134.42	10.38	10/02/2016	2994.8	3020.53	-25.73
23/11/2015	3033.89	3000.11	33.78	31/12/2015	3147.23	3149.39	-2.16	11/02/2016	2978.65	2995.51	-16.86
24/11/2015	3045.7	3035.5	10.20	5/01/2016	3124.26	3149.89	-25.63	12/02/2016	2952.25	2979.04	-26.79
25/11/2015	3027.54	3046.68	-19.14	6/01/2016	3116.28	3125.87	-9.59	16/02/2016	3022.6	2951.98	70.62
27/11/2015	3043.44	3029.48	13.96	7/01/2016	3091.83	3119.42	-27.59	17/02/2016	3027.67	3023.01	4.66
30/11/2015	3043.01	3045.9	-2.89	8/01/2016	3064.32	3094.41	-30.09	18/02/2016	3040.37	3028.37	12.00
1/12/2015	3064.57	3044.65	19.92	11/01/2016	3036.81	3065.84	-29.03	19/02/2016	3055.61	3041.02	14.59
2/12/2015	3059.35	3068.16	-8.81	12/01/2016	3041.55	3037.59	3.96	22/02/2016	3054.49	3056.48	-1.99
3/12/2015	3050.56	3060.45	-9.89	13/01/2016	3061.22	3042.38	18.84	23/02/2016	3072.5	3055.24	17.26
4/12/2015	3035.2	3053.33	-18.13	14/01/2016	3040.03	3062.57	-22.54	24/02/2016	3099.72	3073.91	25.81

Table 3 continued

Table 3 *Continue* Model 1: HWES (alpha, beta) on NZX50 Index Actual vs Prediction											
Observation	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**
7/12/2015	3020.06	3037.61	-17.55	15/01/2016	3069.79	3040.88	28.91	25/02/2016	3096.79	3101.62	-4.83
8/12/2015	3005.41	3021.12	-15.71	19/01/2016	3046.9	3070.7	-23.80	26/02/2016	3096.65	3098.42	-1.77
9/12/2015	3012.51	3006.33	6.18	20/01/2016	3041.68	3047.7	-6.02	29/02/2016	3096.11	3097.85	-1.74
10/12/2015	3006.05	3013.36	-7.31	21/01/2016	3025.35	3042.49	-17.14	1/03/2016	3119.65	3098.23	21.42
11/12/2015	3020.67	3007.09	13.58	22/01/2016	3045.61	3025.91	19.70	2/03/2016	3132.93	3120.74	12.19
3/03/2016	3166.57	3136.28	30.29	11/04/2016	3295.29	3301.13	-5.84	17/05/2016	3415.36	3387.92	27.44
4/03/2016	3185.06	3171.3	13.76	12/04/2016	3295.74	3298.75	-3.01	18/05/2016	3419.17	3416.47	2.70
7/03/2016	3185.46	3186.5	-1.04	13/04/2016	3321.17	3299.58	21.59	19/05/2016	3380.48	3420.15	-39.67
8/03/2016	3198.93	3189.31	9.62	14/04/2016	3343.66	3324.55	19.11	20/05/2016	3383.53	3383.67	-0.14
9/03/2016	3202.4	3200.42	1.98	15/04/2016	3353.91	3348.11	5.80	23/05/2016	3382.51	3386.01	-3.50
10/03/2016	3217.93	3204.81	13.12	18/04/2016	3357.08	3357.39	-0.31	24/05/2016	3365.31	3385.62	-20.31
11/03/2016	3221.46	3224.8	-3.34	19/04/2016	3367.78	3361.98	5.80	25/05/2016	3382.06	3367.69	14.37
14/03/2016	3246.88	3222.76	24.12	20/04/2016	3381.61	3372.5	9.11	26/05/2016	3397.82	3383.65	14.17
15/03/2016	3251.45	3250.76	0.69	21/04/2016	3383.98	3384.87	-0.89	27/05/2016	3419.21	3400.05	19.16
16/03/2016	3243.12	3257.34	-14.22	22/04/2016	3364.38	3389.7	-25.32	31/05/2016	3440.32	3421.25	19.07
17/03/2016	3237.63	3245.26	-7.63	26/04/2016	3329.89	3367.68	-37.79	1/06/2016	3429.82	3442.17	-12.35
18/03/2016	3262.28	3240.85	21.43	27/04/2016	3307.69	3331.67	-23.98	2/06/2016	3420.2	3431.24	-11.04
21/03/2016	3269.99	3266.01	3.98	28/04/2016	3327.08	3310.46	16.62	3/06/2016	3430.58	3423.24	7.34
22/03/2016	3280.98	3271.09	9.89	29/04/2016	3342.08	3328.86	13.22	7/06/2016	3436.89	3432.33	4.56
23/03/2016	3274.19	3285.94	-11.75	2/05/2016	3327.98	3343.97	-15.99	8/06/2016	3413.78	3440.59	-26.81
24/03/2016	3271.09	3277.64	-6.55	3/05/2016	3353.07	3331.12	21.95	9/06/2016	3400.46	3415.21	-14.75
29/03/2016	3277.86	3275.44	2.42	4/05/2016	3343.99	3354.97	-10.98	10/06/2016	3401.06	3402.19	-1.13
30/03/2016	3292.95	3279.36	13.59	5/05/2016	3369.47	3345.58	23.89	14/06/2016	3333.35	3403.71	-70.36
31/03/2016	3310.02	3299.14	10.88	6/05/2016	3380.06	3373.44	6.62	15/06/2016	3350.23	3334.55	15.68
1/04/2016	3287.81	3311.6	-23.79	9/05/2016	3372.89	3383.99	-11.10	16/06/2016	3356.92	3351.54	5.38
4/04/2016	3305.25	3289.72	15.53	10/05/2016	3384.82	3376.58	8.24	17/06/2016	3336.7	3358.39	-21.69
5/04/2016	3291.63	3308.48	-16.85	11/05/2016	3401.59	3386.22	15.37	20/06/2016	3347.65	3338.25	9.40
6/04/2016	3300.68	3295.72	4.96	12/05/2016	3390.05	3404.12	-14.07	21/06/2016	3332.96	3347.78	-14.82
7/04/2016	3310.05	3304.29	5.76	13/05/2016	3386.82	3393.25	-6.43	22/06/2016	3304.86	3334.23	-29.37
8/04/2016	3297.83	3311.86	-14.03	16/05/2016	3385.62	3389.41	-3.79	23/06/2016	3324.17	3306.06	18.11
24/06/2016	3249.33	3323.73	-74.40	1/08/2016	3583.9	3588.47	-4.57	6/09/2016	3646.47	3615.59	30.88
27/06/2016	3258.66	3249.67	8.99	2/08/2016	3570.53	3587.14	-16.61	7/09/2016	3677.5	3650.24	27.26
28/06/2016	3273.11	3258.96	14.15	3/08/2016	3545.3	3575.3	-30.00	8/09/2016	3652.94	3681.2	-28.26
29/06/2016	3315.81	3272.68	43.13	4/08/2016	3555.38	3548.7	6.68	9/09/2016	3620.88	3657.01	-36.13
30/06/2016	3361.29	3316.78	44.51	5/08/2016	3560.41	3558.03	2.38	12/09/2016	3528.9	3624.13	-95.23
1/07/2016	3375.28	3363.13	12.15	8/08/2016	3579.84	3565.16	14.68	13/09/2016	3511.15	3530.88	-19.73
5/07/2016	3397.09	3377.28	19.81	9/08/2016	3587.08	3582.42	4.66	14/09/2016	3491.91	3512.87	-20.96
6/07/2016	3400.13	3398.21	1.92	10/08/2016	3580.48	3589.91	-9.43	15/09/2016	3483.36	3492.53	-9.17
7/07/2016	3414.64	3401.64	13.00	11/08/2016	3582.53	3585.6	-3.07	16/09/2016	3509.63	3481.79	27.84

Table 3 continued

Table 3 *Continue* Model 1: HWES (alpha, beta) on NZX50 Index Actual vs Prediction											
Observation	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**
8/07/2016	3411.03	3416.72	-5.69	12/08/2016	3587.05	3585.99	1.06	19/09/2016	3522.99	3510.43	12.56
11/07/2016	3441.41	3413.45	27.96	15/08/2016	3599.35	3588.49	10.86	20/09/2016	3537.67	3523.39	14.28
12/07/2016	3449.7	3446.04	3.66	16/08/2016	3561.51	3604.46	-42.95	21/09/2016	3524.47	3538.06	-13.59
13/07/2016	3442.32	3453.37	-11.05	17/08/2016	3583.11	3563.42	19.69	22/09/2016	3520.78	3526.34	-5.56
14/07/2016	3449.29	3447.04	2.25	18/08/2016	3597.78	3586.92	10.86	23/09/2016	3513.57	3521.05	-7.48
15/07/2016	3445.67	3451.92	-6.25	19/08/2016	3607.59	3601.95	5.64	26/09/2016	3498.22	3514.97	-16.75
18/07/2016	3461.78	3447.59	14.19	22/08/2016	3634.97	3609.19	25.78	27/09/2016	3492.47	3497.81	-5.34
19/07/2016	3485.59	3464.63	20.96	23/08/2016	3637.49	3639.58	-2.09	28/09/2016	3506.94	3493.35	13.59
20/07/2016	3494.28	3490.72	3.56	24/08/2016	3609.71	3641.85	-32.14	29/09/2016	3525.23	3508.17	17.06
21/07/2016	3514.44	3496.7	17.74	25/08/2016	3617.98	3611.39	6.59	30/09/2016	3533.7	3526.91	6.79
22/07/2016	3520.29	3518.75	1.54	26/08/2016	3600.21	3620.8	-20.59	3/10/2016	3539.17	3534.74	4.43
25/07/2016	3564.74	3524.55	40.19	29/08/2016	3588.5	3603.7	-15.20	4/10/2016	3529.56	3541.02	-11.46
26/07/2016	3561.37	3571.41	-10.04	30/08/2016	3598.58	3591.41	7.17	5/10/2016	3490.53	3531.33	-40.80
27/07/2016	3557.24	3565.33	-8.09	31/08/2016	3603.39	3601.92	1.47	6/10/2016	3455.07	3491.35	-36.28
28/07/2016	3559.4	3565.75	-6.35	1/09/2016	3611.98	3606.76	5.22	7/10/2016	3440.92	3456.33	-15.41
29/07/2016	3579.76	3561.25	18.51	2/09/2016	3613.4	3615.22	-1.82	10/10/2016	3416.49	3442.01	-25.52
11/10/2016	3420	3416.84	3.16	16/11/2016	3272.8	3243.21	29.59	22/12/2016	3273.33	3249.82	23.51
12/10/2016	3411.94	3420.27	-8.33	17/11/2016	3268.04	3272.52	-4.48	23/12/2016	3285.33	3271.77	13.56
13/10/2016	3417.99	3412.14	5.85	18/11/2016	3288.76	3268.64	20.12	28/12/2016	3284.75	3283.85	0.90
14/10/2016	3424.33	3414.26	10.07	21/11/2016	3284.49	3286.52	-2.03	29/12/2016	3292.63	3283.32	9.31
17/10/2016	3392.22	3425.04	-32.82	22/11/2016	3268.88	3285.06	-16.18	30/12/2016	3287.35	3292.9	-5.55
18/10/2016	3347.44	3392.53	-45.09	23/11/2016	3285.09	3265.92	19.17	4/01/2017	3331.82	3287.45	44.37
19/10/2016	3349.09	3343.87	5.22	25/11/2016	3304.38	3285.84	18.54	5/01/2017	3332.03	3331.7	0.33
20/10/2016	3347.77	3348.17	-0.40	28/11/2016	3305.98	3303.94	2.04	6/01/2017	3329.38	3331.67	-2.29
21/10/2016	3340.39	3346.56	-6.17	29/11/2016	3305.4	3304.22	1.18	9/01/2017	3349.48	3329.76	19.72
25/10/2016	3361.73	3334.56	27.17	30/11/2016	3302.26	3303.28	-1.02	10/01/2017	3361.35	3350	11.35
26/10/2016	3310.53	3359.77	-49.24	1/12/2016	3317.32	3301.68	15.64	11/01/2017	3376.64	3361.8	14.84
27/10/2016	3332.49	3309.86	22.63	2/12/2016	3303.68	3316.19	-12.51	12/01/2017	3373.49	3377.2	-3.71
28/10/2016	3333.14	3328.95	4.19	5/12/2016	3277.82	3302.66	-24.84	13/01/2017	3365.55	3374.01	-8.46
31/10/2016	3341.48	3331.6	9.88	6/12/2016	3301.89	3275.05	26.84	17/01/2017	3373.19	3366.23	6.96
1/11/2016	3326.99	3340.23	-13.24	7/12/2016	3292.05	3300.63	-8.58	18/01/2017	3371.43	3373.97	-2.54
2/11/2016	3289.68	3322.5	-32.82	8/12/2016	3303.97	3290.94	13.03	19/01/2017	3372.9	3371.88	1.02
3/11/2016	3253.77	3290.47	-36.70	9/12/2016	3293.12	3303.01	-9.89	20/01/2017	3366.27	3373.71	-7.44
4/11/2016	3219.95	3248.68	-28.73	12/12/2016	3284.87	3293.78	-8.91	23/01/2017	3375.52	3366.81	8.71
7/11/2016	3298.57	3219.02	79.55	13/12/2016	3272.53	3285.25	-12.72	24/01/2017	3373.76	3375.99	-2.23
8/11/2016	3309.17	3294.59	14.58	14/12/2016	3247.53	3273.86	-26.33	25/01/2017	3386.54	3374.05	12.49
9/11/2016	3198.71	3304.84	-106.13	15/12/2016	3224	3247.56	-23.56	27/01/2017	3407.24	3388.66	18.58
10/11/2016	3232.07	3192.23	39.84	16/12/2016	3229.56	3224.62	4.94	30/01/2017	3383.98	3407.95	-23.97
11/11/2016	3213.92	3229.72	-15.80	19/12/2016	3241.98	3225.94	16.04	31/01/2017	3367.36	3384.38	-17.02

Table 3 continued

Table 3 *Continue* Model 1: HWES (alpha, beta) on NZX50 Index Actual vs Prediction											
Observation	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**
14/11/2016	3231.17	3211.4	19.77	20/12/2016	3243.61	3241.28	2.33	1/02/2017	3369.63	3368.13	1.50
15/11/2016	3246.83	3227.08	19.75	21/12/2016	3249.86	3243.11	6.75	2/02/2017	3368.69	3370.98	-2.29
3/02/2017	3388.2	3368.97	19.23	14/03/2017	3409.22	3420.32	-11.10	20/04/2017	3383.74	3398.47	-14.73
7/02/2017	3375.14	3388.72	-13.58	15/03/2017	3386.57	3409.8	-23.23	21/04/2017	3388.05	3384.22	3.83
8/02/2017	3374.77	3375.79	-1.02	16/03/2017	3385.74	3387.2	-1.46	24/04/2017	3400.16	3388.58	11.58
9/02/2017	3401.21	3375.24	25.97	17/03/2017	3388.66	3386.43	2.23	26/04/2017	3452.98	3400.53	52.45
10/02/2017	3393	3402	-9.00	20/03/2017	3335.37	3389.59	-54.22	27/04/2017	3462.15	3454.45	7.70
13/02/2017	3407.83	3393.86	13.97	21/03/2017	3348.84	3333.29	15.55	28/04/2017	3473.51	3466.25	7.26
14/02/2017	3412.69	3408.73	3.96	22/03/2017	3337.16	3349.3	-12.14	1/05/2017	3475.14	3474.54	0.60
15/02/2017	3426.59	3413.4	13.19	23/03/2017	3331.38	3336.05	-4.67	2/05/2017	3494.1	3476.15	17.95
16/02/2017	3388.39	3427.48	-39.09	24/03/2017	3336.7	3332.72	3.98	3/05/2017	3486.26	3495.21	-8.95
17/02/2017	3385.31	3388.78	-3.47	27/03/2017	3331.46	3336.77	-5.31	4/05/2017	3473.35	3487.33	-13.98
21/02/2017	3395.89	3385.66	10.23	28/03/2017	3332.64	3330.85	1.79	5/05/2017	3467.27	3474.78	-7.51
22/02/2017	3370.49	3396.47	-25.98	29/03/2017	3362.35	3333.34	29.01	8/05/2017	3495.19	3468.5	26.69
23/02/2017	3383.4	3370.63	12.77	30/03/2017	3376.38	3362.77	13.61	9/05/2017	3488.44	3497.04	-8.60
24/02/2017	3368.24	3384.21	-15.97	31/03/2017	3388.9	3376.68	12.22	10/05/2017	3494.12	3490.56	3.56
27/02/2017	3374.71	3368.46	6.25	3/04/2017	3402.2	3389.97	12.23	11/05/2017	3524.96	3495.39	29.57
28/02/2017	3416.79	3375.35	41.44	4/04/2017	3410.33	3402.7	7.63	12/05/2017	3507.39	3526.79	-19.40
1/03/2017	3407.28	3417.61	-10.33	5/04/2017	3419.99	3410.89	9.10	15/05/2017	3496.83	3509.15	-12.32
2/03/2017	3417.35	3407.94	9.41	6/04/2017	3431.51	3420.99	10.52	16/05/2017	3486.32	3497.99	-11.67
3/03/2017	3410.22	3418.47	-8.25	7/04/2017	3409.96	3432.4	-22.44	17/05/2017	3492.99	3488.17	4.82
6/03/2017	3418.73	3411.27	7.46	10/04/2017	3407.66	3410.71	-3.05	18/05/2017	3467.96	3494.86	-26.90
7/03/2017	3413.45	3419.46	-6.01	11/04/2017	3414.96	3408.4	6.56	19/05/2017	3477.53	3469.05	8.48
8/03/2017	3417	3414.12	2.88	12/04/2017	3413.63	3415.7	-2.07	22/05/2017	3485.71	3478.91	6.80
9/03/2017	3394.4	3417.68	-23.28	13/04/2017	3403.39	3414.66	-11.27	23/05/2017	3474.05	3486.87	-12.82
10/03/2017	3411.25	3395.05	16.20	18/04/2017	3405.18	3403.97	1.21	24/05/2017	3490.89	3475	15.89
13/03/2017	3419.24	3411.98	7.26	19/04/2017	3398.08	3406.17	-8.09	25/05/2017	3492.98	3491.95	1.03
26/05/2017	3496.31	3494.54	1.77	6/07/2017	3571.44	3557.71	13.73	10/08/2017	3645.51	3652.23	-6.72
30/05/2017	3482.4	3497.5	-15.10	7/07/2017	3567.94	3573.66	-5.72	11/08/2017	3612.47	3648.03	-35.56
31/05/2017	3485.66	3483.43	2.23	10/07/2017	3550.07	3569.13	-19.06	14/08/2017	3632.51	3614.18	18.33
1/06/2017	3498.05	3486.51	11.54	11/07/2017	3570.96	3550.98	19.98	15/08/2017	3656.76	3633.29	23.47
2/06/2017	3521.09	3499.7	21.39	12/07/2017	3551.04	3572.15	-21.11	16/08/2017	3675.29	3658.1	17.19
6/06/2017	3516.89	3522.65	-5.76	13/07/2017	3561.83	3552.2	9.63	17/08/2017	3683.11	3678.19	4.92
7/06/2017	3503.02	3518.39	-15.37	14/07/2017	3580.02	3563.43	16.59	18/08/2017	3684.74	3686.42	-1.68
8/06/2017	3495.98	3504.05	-8.07	17/07/2017	3603.32	3581.5	21.82	21/08/2017	3682.05	3687.19	-5.14
9/06/2017	3485.4	3497.16	-11.76	18/07/2017	3606.96	3606.56	0.40	22/08/2017	3681.39	3683.5	-2.11
13/06/2017	3486.72	3486.3	0.42	19/07/2017	3618.85	3609.41	9.44	23/08/2017	3687.18	3683.38	3.80
14/06/2017	3506.77	3487.64	19.13	20/07/2017	3590.63	3620.36	-29.73	24/08/2017	3682.01	3688.95	-6.94
15/06/2017	3519.23	3508.32	10.91	21/07/2017	3589.89	3591.93	-2.04	25/08/2017	3677.05	3683.36	-6.31

Table 3 continued

Table 3 *Continue* Model 1: HWES (alpha, beta) on NZX50 Index Actual vs Prediction											
Observation	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**	**Observation**	**Actual (test)**	**Predicted**	**Residual**
16/06/2017	3536.27	3520.07	16.20	24/07/2017	3595.24	3591.83	3.41	28/08/2017	3662.2	3678.81	-16.61
19/06/2017	3554.66	3537.46	17.20	25/07/2017	3609.64	3596.69	12.95	29/08/2017	3620.78	3664.38	-43.60
20/06/2017	3552.09	3556.98	-4.89	26/07/2017	3608.48	3611.35	-2.87	30/08/2017	3633.51	3622.03	11.48
21/06/2017	3524.26	3553.64	-29.38	27/07/2017	3609.01	3610.04	-1.03	31/08/2017	3649.94	3634.99	14.95
22/06/2017	3541.39	3525.34	16.05	28/07/2017	3575.22	3611.39	-36.17	1/09/2017	3652.06	3651.55	0.51
23/06/2017	3536.69	3542.53	-5.84	31/07/2017	3600.71	3576.37	24.34	5/09/2017	3629.2	3653.86	-24.66
26/06/2017	3556.28	3537.77	18.51	1/08/2017	3617.31	3602.14	15.17	6/09/2017	3633.08	3630.36	2.72
27/06/2017	3570.23	3558.13	12.10	2/08/2017	3626.14	3619.83	6.31	7/09/2017	3634.6	3634.56	0.04
28/06/2017	3569.36	3572.98	-3.62	3/08/2017	3628.68	3627.13	1.55	8/09/2017	3656.37	3635.65	20.72
29/06/2017	3597.89	3572.52	25.37	4/08/2017	3625.11	3631.09	-5.98	11/09/2017	3656.48	3657.78	-1.30
30/06/2017	3563.25	3601.11	-37.86	7/08/2017	3637.02	3626.2	10.82	12/09/2017	3651.2	3658.29	-7.09
3/07/2017	3552.48	3564.3	-11.82	8/08/2017	3642.24	3639.75	2.49	13/09/2017	3641.77	3652.53	-10.76
5/07/2017	3555.86	3554.06	1.80	9/08/2017	3650.16	3644.31	5.85	14/09/2017	3635.98	3643.15	-7.17
15/09/2017	3609.67	3637.36	-27.69	20/10/2017	3748.92	3748.09	0.83	28/11/2017	3746.72	3765.03	-18.31
18/09/2017	3607.84	3610.77	-2.93	24/10/2017	3749.18	3751.59	-2.41	29/11/2017	3745.85	3748.54	-2.69
19/09/2017	3610.54	3608.5	2.04	25/10/2017	3745.75	3750.72	-4.97	30/11/2017	3764.64	3747.61	17.03
20/09/2017	3635.6	3611.65	23.95	26/10/2017	3729.18	3747.52	-18.34	1/12/2017	3765.27	3766.54	-1.27
21/09/2017	3609.69	3636.89	-27.20	27/10/2017	3728.37	3731.28	-2.91	4/12/2017	3761.42	3766.14	-4.72
22/09/2017	3618.66	3610.68	7.98	30/10/2017	3755.58	3730.31	25.27	5/12/2017	3754.65	3763.61	-8.96
25/09/2017	3641.96	3619.03	22.93	31/10/2017	3756.66	3757.44	-0.78	6/12/2017	3733.25	3756.41	-23.16
26/09/2017	3650.02	3643.23	6.79	1/11/2017	3713.99	3758.6	-44.61	7/12/2017	3751.74	3734.54	17.20
27/09/2017	3662.33	3651.47	10.86	2/11/2017	3727.48	3715.59	11.89	8/12/2017	3780.42	3752.84	27.58
28/09/2017	3655.03	3663.68	-8.65	3/11/2017	3719.21	3729.26	-10.05	11/12/2017	3799.9	3782.54	17.36
29/09/2017	3662.77	3656.53	6.24	6/11/2017	3713.91	3720.55	-6.64	12/12/2017	3801.41	3803.18	-1.77
2/10/2017	3661.52	3663.93	-2.41	7/11/2017	3712.09	3715.21	-3.12	13/12/2017	3803.14	3803.61	-0.47
3/10/2017	3663.61	3663.07	0.54	8/11/2017	3707.82	3713.12	-5.30	14/12/2017	3821.13	3805.75	15.38
4/10/2017	3671.13	3665.17	5.96	9/11/2017	3698.44	3709.28	-10.84	15/12/2017	3838.16	3826.32	11.84
5/10/2017	3678.57	3672.79	5.78	10/11/2017	3675.94	3699.7	-23.76	18/12/2017	3830.49	3841.28	-10.79
6/10/2017	3678.73	3679.96	-1.23	13/11/2017	3674.92	3676.86	-1.94	19/12/2017	3856.68	3832.34	24.34
9/10/2017	3693.92	3681.55	12.37	14/11/2017	3689.47	3673.01	16.46	20/12/2017	3848.58	3859.46	-10.88
10/10/2017	3706.62	3697.74	8.88	15/11/2017	3685.75	3690.3	-4.55	21/12/2017	3839.81	3852.75	-12.94
11/10/2017	3717.3	3711.15	6.15	16/11/2017	3701.77	3686.31	15.46	22/12/2017	3854.49	3842.71	11.78
12/10/2017	3720.59	3722.32	-1.73	17/11/2017	3713.74	3703.05	10.69	27/12/2017	3845.31	3857.09	-11.78
13/10/2017	3730.37	3723.23	7.14	20/11/2017	3726.56	3715.07	11.49	28/12/2017	3860.05	3847.72	12.33
16/10/2017	3731.02	3731.99	-0.97	21/11/2017	3725.95	3727.97	-2.02	29/12/2017	3855.25	3862.75	-7.50
17/10/2017	3740.85	3734.23	6.62	22/11/2017	3733.55	3727.35	6.20				
18/10/2017	3742.22	3743.64	-1.42	24/11/2017	3742.96	3735.2	7.76				
19/10/2017	3746.4	3744.68	1.72	27/11/2017	3762.65	3744.32	18.33				

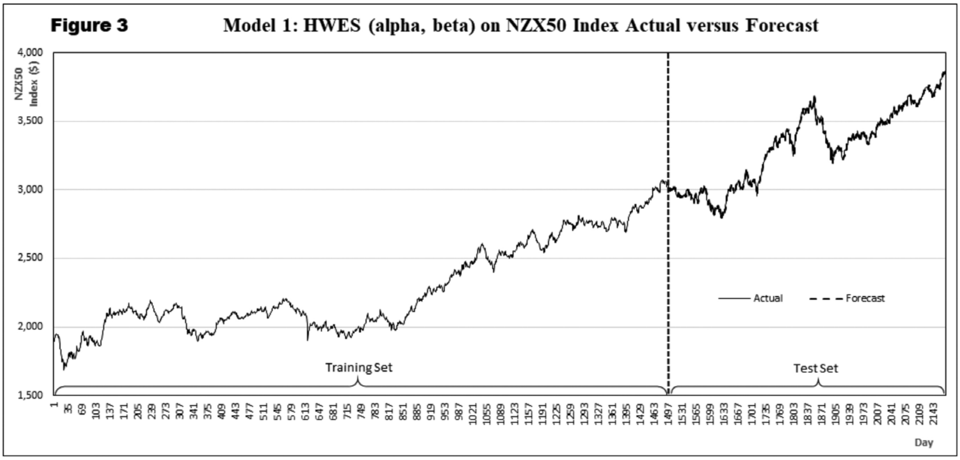
One sample hypothesis testing on the residuals are statistically insignificant and these results are presented in Table 4.

Table 4

Table 4 t-Test: One Sample	
	Residual
Mean	0.1760
Variance	339.3597
Observations	672
Hypothesized Mean	0
df	671
t Stat	0.2476
P(T<=t) one-tail	0.4022
t Critical one-tail	1.6471
P(T<=t) two-tail	0.8045
t Critical two-tail	1.9635

Predicted values generated from Model 1 and the actual S&P/NZX50 Index values are presented in the "Test Set" section of Figure 3. The "dashed-line" in Figure 3 shows the predicted values from Model 1 whilst the "solid-line" shows the actual values. The figure represents quite a strong forecasting accuracy of the S&P/NZX50 Index based on Model 1 [HWES (alpha, beta) on the NZX50 Index].

Figure 3



The predictive model 1 [HWES (alpha, beta) on NZX50 Index] suggests that, on average, S&P/NZX50 Index increases each day (much like a random walk with drift). Further, the change in index price is affected by the previous day's change. If the previous day's price change was positive (negative), this effect will be positive (negative) but smaller in magnitude than that change.

6. CONCLUSIONS

Taking a technical analysis perspective, this study employs the Holt-Winters Exponential Smoothing (HWES) methodology to predict the New Zealand stock market (S&P/NZX50) Index. Multiple performance evaluation measures, namely MAE, MAPE, and RMSE are used with AIC, ADF, and Phillips-Perron unit root tests. The results of the performance evaluation measures reinforce that model 1 [HWES (alpha, beta) on NZX50 Index] outperforms model 2 [HWES (alpha) on the difference of NZX50 Index].

Applying model 1 [HWES (alpha, beta) on NZX50 Index] for prediction purposes we find that the predictions are very accurate of forecasting the next lags of the S&P/NZX50 Index. We find that the S&P/NZX50 Index follows more or less a pattern of a random walk with drift. An increase (decrease) of the index would usually be followed by a marginally smaller impact [smaller increase (decrease)].

The HWES models evaluated in this paper were specifically trained to capture the smoothed level, trend, and seasonal components inherent with the NZX 50 index for the period of 2009 to 2015 (having 1,500 observations for training, 70%). The trained HWES models were then used to forecast the NZX 50 index from 2016 to 2017 (673 observations in total for prediction, 30%). The forecasting results of model 1 [HWES (alpha, beta) on the NZX50 Index] demonstrates its predictive efficiency and effectiveness.

The proposed model could be successfully implemented in forecasting other stock market time series or same index for different periods (windows) if effective and substantiate algorithm training is carried out. A potential future research endeavour could be to compare and contrast the predictive effectiveness of HWES with a deep-learning model such as long short term memory (LSTM) which has the calibre to remember and efficiently learn the long-term dependencies.

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REFERENCES

Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle.

Archibald, B. C. (1990). Parameter space of the Holt-Winters' model. *International Journal of Forecasting, 6*(2), 199-209.

Assimakopoulos,V., & Nikolopoulos, K. (2000). The theta model: a decomposition approach to forecasting. *International Journal of Forecasting*, 16, 521–530.

Balaban, E., Bayar, A., & Faff, R. W. (2006). Forecasting stock market volatility: Further international evidence. *The European Journal of Finance, 12*(2), 171-188.

Bley, J., & Olson, D. (2008). Volatility forecasting performance with VIX: Implied volatility versus historical standard deviation and conditional volatility. *Advances in Financial Planning and Forecasting*, (3), 67-91.

Brown, R. G. (1959). *Statistical forecasting for inventory control.* McGraw/Hill.

Brown, R. G. (1963). *Smoothing, forecasting and prediction of discrete time series.* Englewood Cliffs, NJ7 Prentice-Hall.

Carreno, J., & Madinaveitia, J. (1990). A modification of time series forecasting methods for handling announced price increases. *International Journal of Forecasting, 6*(4), 479-484.

Chen, N. F. (1991). Financial investment opportunities and the macroeconomy. *The Journal of Finance, 46*(2), 529-554.

Chen, N. F., Roll, R., & Ross, S. A. (1986). Economic forces and the stock market. Journal of business, 383-403.

Covel, M. (2004). *Trend following: how great traders make millions in up or down markets*. FT Press.

Dassanayake, W., & Jayawardena, C. (2017, January). Determinants of stock market index movements: Evidence from New Zealand stock market. In *2017 6th National Conference on Technology and Management (NCTM)* (pp. 6-11). IEEE.

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association, 74*(366a), 427-431.

Dickey, D. A., & Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica: journal of the Econometric Society*, 1057-1072.

Engle, R. F., & Granger, C. W. (1987). Co-integration and error correction: representation, estimation, and testing. *Econometrica: journal of the Econometric Society*, 251-276.

Fama, E. F. (1991). Efficient capital markets: II. *The journal of finance, 46*(5), 1575-1617.

Faraway, J., & Chatfield, C. (1998). Time series forecasting with neural networks: a comparative study using the airline data. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*. 47(2), 231-250.

Gardner Jr, E. S. (2006). Exponential smoothing: The state of the art—Part II. *International journal of forecasting, 22*(4), 637-666.

Granger, C. J. (1986). Developments in the study of cointegrated economic variables. *Oxford Bulletin of economics and statistics, 48*(3), 213-228.

Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. *Neural computation.* 9(8), 1735-1780

Holt, C. C. (1957). Forecasting seasonals and trends by exponentially weighted moving averages, Office of Naval Research. *Research memorandum, 52*.

Holt, C. C. (2004). Forecasting seasonals and trends by exponentially weighted moving averages. *International journal of forecasting, 20(*1), 5-10.

Holt, C. C., Modigliani, F., Muth, J. F., & Simon, H. A. (1960). Planning production, inventories, and work force. Englewood Cliffs, NJ7 Prentice-Hall.

Hyndman, R. J., & Billah, B. (2003). Unmasking the Theta method. *International Journal of Forecasting*, 19, 287– 290.

Hyndman, R. J., Koehler, A. B., Snyder, R. D., & Grose, S. (2002). A state-space framework for automatic forecasting using exponential smoothing methods. International Journal of forecasting, 18(3), 439-454.

Kihoro, J., Otieno R., & C. Wafula. (2004). Seasonal time series forecasting: a comparative study of ARIMA and ANN models. *African Journal of Science and Technology.* 5 (2).

Kwon, C. S., & Shin, T. S. (1999). Cointegration and causality between macroeconomic variables and stock market returns. *Global finance journal, 10*(1), 71-81.

Lawton, R. (1998). How should additive Holt-Winters estimates be corrected? *International Journal of Forecasting, 14*(3), 393-403.

Leung, M. T., Daouk, H., & Chen, A. S. (2000). Forecasting stock indices: a comparison of classification and level estimation models. *International Journal of Forecasting, 16*(2), 173-190.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of applied econometrics, 11*(6), 601-618.

Makridakis, S. (1993). Accuracy measures: theoretical and practical concerns. *International Journal of Forecasting, 9*(4), 527-529.

Maris, K., Pantou, G., Nikolopoulos, K., Pagourtzi, E., & Assimakopoulos, V. (2004). A study of financial volatility forecasting techniques in the FTSE/ASE 20 index. *Applied Economics Letters, 11*(7), 453-457.

McKenzie, E. (1986). Error analysis for Winters additive seasonal forecasting system. *International Journal of Forecasting* 2, no. 3 (1986): 373-382.

Muth, J. F. (1960). Optimal properties of exponentially weighted forecasts. *Journal of the American statistical association, 55*(290), 299-306.

Ord, J. K., Koehler, A. B., & Snyder, R. D. (1997). Estimation and prediction for a class of dynamic nonlinear statistical models. *Journal of the American Statistical Association, 92*(440), 1621-1629.

Pegels, C. C. (1969). Exponential forecasting: some new variations. *Management Science*, 311-315.

Pereira, R. (2004). Forecasting Portuguese stock market volatility. J*ournal of Forecasting, 18*, 333-343.

Phillips, P. (1987). Time Series Regression with a Unit Root. *Econometrica, 55*(2), 277-301. doi:10.2307/1913237

Phillips, P. C., & Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika, 75*(2), 335-346.

Poon, S. H., Hyung, N., & Granger, C. W. (2006). A source of long memory in volatility. *Available at SSRN 904582*.

Proceedings of IEEE international symposium on information theory. Awajan, A. M., Ismail, M. T., & Wadi, S. A. (2018). Improving forecasting accuracy for stock market data using EMD-HW bagging. *PloS one, 13*(7).

Roberts, S. A. (1982). A general class of Holt-Winters type forecasting models. Management Science, 28(7), 808-820.

Rosas, A. L., & Guerrero, V. M. (1994). Restricted forecasts using exponential smoothing techniques. *International Journal of Forecasting, 10*(4), 515-527.

Schwager, J. D. (1993). *Market wizards: Interviews with top traders.* Collins, New York.

Schwager, J. D. (1995). *The new market wizards: Conversations with America's top traders*. Wiley, New York.

Sharif, O., & Hasan, M. Z. (2019). Forecasting the Stock Price by using Holt's Method. *Indonesian Journal of Contemporary Management Research, 1*(1), 15-24.

S&P/NZX50 Index. (2018). S&P Dow Jones Indices. https://www.spglobal.com/spdji/en/indices/equity/sp-nzx-50-index/#overview.

Statistics, O. E. C. D. (2013). Glossary of Statistical Terms. *Competitiveness in international trade. [www] http://stats. OECD. org/glossary/detail. asp*.

Taylor, J. W. (2004). Volatility forecasting with smooth transition exponential smoothing. *International Journal of Forecasting, 20*(2), 273-286.

Tseng, K. C., Kwon, O., & Tjing, L. C. (2012). Time series and neural network forecasts of daily stock prices. *Investment Management and Financial Innovations,* (9, Iss. 1), 32-54.

Williams, D. W., & Miller, D. (1999). Level-adjusted exponential smoothing for modeling planned discontinuities. *International Journal of Forecasting, 15*(3), 273-289.

Winters, P. R. (1960). Forecasting sales by exponentially weighted moving averages. *Management Science, 6*(3), 324-342.

Wongbangpo, P., & Sharma, S. C. (2002). Stock market and macroeconomic fundamental dynamic interactions: ASEAN-5 countries. *Journal of Asian Economics, 13*(1), 27-51.

Yu, J. (2002). Forecasting volatility in the New Zealand stock market. *Applied Financial Economics, 12*(3), 193-202.